# STEP BY STEP

## MATHS FOR ALL


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STEP – 3

# ABOUT THE BOOK

Everything around us can be understood better with Mathematics as it can help children to think about many aspects of their world through its connection with them.

For students, learning usually happens in the best way when they can relate it to real life situations. With each higher class, it becomes more advanced and challenging. Many students find it difficult to understand the abstract mathematical concepts and have to work harder and practice longer for this.

However, by infusing real life examples with mathematical concepts, teachers can help students view mathematics from an entirely different point of view.

The booklet **''STEP BY STEP"** fulfils the objective that concepts in Mathematics can be learnt in a joyful manner. It will also enhance the CCT skill of learning.

We are thankful to the **DIRECTOR OF SCHOOL EDUCATION, SH. RUBINDERJIT SINGH BRAR** for his guidance and motivation which helped us to complete this task and we hope that students will also get benefit from this booklet.

SAMEER  **LOGARITHMS**  SIYA

**Hello students, wish you all a**

**VERY HAPPY NEW YEAR 2021.**

**In our previous booklet "Step 2". We asked you a question regarding Takkas. If you remember the last part of that question was** "How many Takkas will be there on square 20? On square 30? On square 64? Write all these in exponential form"

**Now can any one of you tell me exactly the number of takkas on Square 64 ?**

In exponential form its answer is $2^{63}$

& $2^{63}$ = 2 x 2 x 2 x 2....................63 times

2 ko 63 times kaise multiply karein. Its very large. Aao teacher se poochte hain.

This is very large number approximately 19 digit number. Bacho we calculate such type of large number with the help of another mathematical tool called **LOGARITHMS**

Logarithms, Ye kya hota hai, ye to humne kabhi suna hi nhi .

**Daro mat bacho.**

This is very interesting, I will introduce this concept to you as it is used to calculate very large numbers in a very easy way

| EXPONENTIAL | | LOGARITHMIC | |
|---|---|---|---|
| Form | Read as | Form | Read as |
| $2^3 = 8$ | 2x2x2 = 8 | $\log_2 8 = 3$ | log of 8 to the base 2 is 3 |
| $2^6 = 64$ | 2x2x2x2x2x2 = 64 | $\log_2 64 = 6$ | log of 64 to the base 2 is 6 |
| $3^4 = 81$ | 3x3x3x3 = 81 | $\log_3 81 = 4$ | log of 81 to the base 3 is 4 |

## DEFINITION OF LOGARITHMS

**Exponents and Logarithms are inverses of each other. Any time you want to better understand numbers of very large and very small magnitudes, we make use of logarithms.**

**Thus if in exponential form we have $a^m = x$ thenwe write this in logarithmic form as $\log_a x = m$, herewe actually want to know how many times 'a' should be multiplied to get 'x' and the answer is 'm'**

# PROPERTIES OF LOGARITHMS

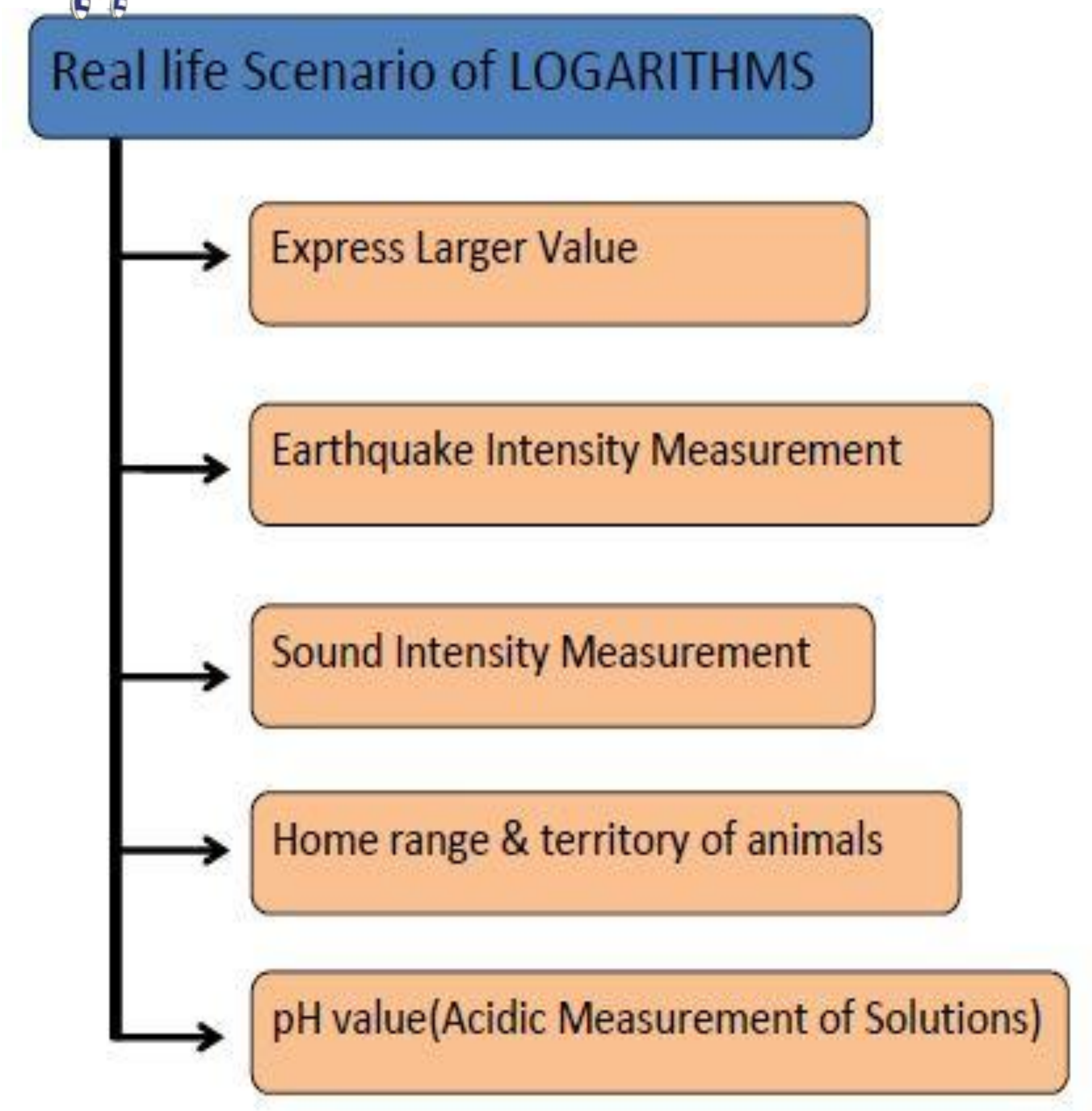
## SEE BELOW

| |
|---|
| $\log_a x^n = n \log_a x$ |
| $\log_a(x.y) = \log_a x + \log_a y$ |
| $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$ |
| $\log_a a = 1 \quad (\text{for } a > 0 \text{ \& } a \neq 1)$ |
| $\log_a 1 = 0$ |
| **All of them can be proved using the definition of 'log' i.e. if $\log_a x = m$ , then $a^m = x$ .In this way convert them into exponential form and then can be proved using exponential rules.** |

There are two types of logarithmic one is called natural logarithmic (developed by John Napier with base 'e') and other practical logarithmic (developed by Henry Briggs with base 10). Now let me explain you the uses of Practical logarithms one by one.

Here we will discuss practical logarithms only.

These are some uses of Logarithms in our day to day life. Logarithms rules are very similar to exponent rule.



# 1. USES OF LOGARITHM TO EXPRESS LARGER VALUES

Anant deposit 100,000/- at bank that pays 12% per year compounded annually, how much amount he gets after


i. 2 years if interest is compounded annually.

ii. 15 years if interest is compounded annually.

**Solution** :- Here P = Rs. 100,000/-, r = 12% = 0.12

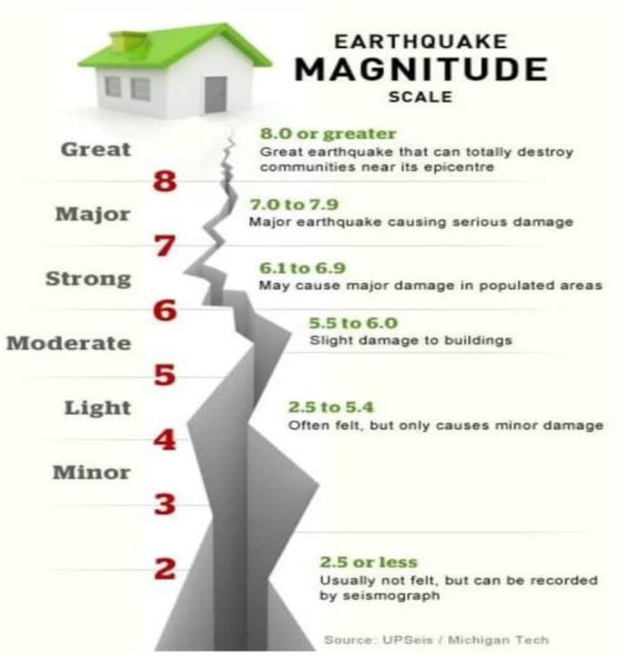
Using the formula : $A = P(1+r)^n$ (you must be remembering compound interest)

we get, $A = 1{,}00{,}000(1+ 0.12)^n = 100000 \times (1.12)^n$

| **Amount after 2 years**<br>**n = 2** | **Amount after 15 years**<br>**n = 15** |
|---|---|
| ➔ $A = 100000\ (1.12)^2$<br>➔ $A = 100000 \times 1.12 \times 1.12$<br>➔ A = Rs. 125440/-<br> | ➔ $A = 100000\ (1.12)^{15}$<br>**Now multiplying 1.12 fifteen times is a very difficult task. Here comes the use of practical logarithmic. See below :-**<br>Taking ' log ' both sides<br>➔ $\log_{10} A = \log_{10}\{10^5\ (1.12)^{15}\}$<br>➔ $\log_{10} A = \log_{10} 10^5 + \log_{10} (1.12)^{15}$<br>Now using properties of 'log'<br>➔ $\log_{10} A = 5 \log_{10} 10 + 15 \log_{10} (1.12)$<br>➔ $\log_{10} A = 5 + 15\ (0.0492)$<br>(Here $\log_{10} 1.12 = 0.0492$ you will study in higher classes)<br>➔ $\log_{10} A = 5 + 15\ (0.0492)$<br>$= 5 + 0.7380 = 5.7380$<br>➔ $A = 10^{5.7380}$<br>➔ A = 547000/- (approx.)<br>(Here again we have used $10^{5.7380} = 547000$ will study in higher classes). |
| **Thus Anant will get Rs. 125440/- after 2 years and Rs 5,47,000/- for his deposit of Rs 1,00,000 after 15 years.** | |

# 2. USES OF LOGARITHM TO MEASURE EARTHQUAKE

An earthquake is the result of a **sudden release of energy in the Earth's crust** that creates vibrations or seismic waves that radiate upto the surface, causing the ground to shake. Instruments called **SEISMOGRAPHS** detect movement in the earth due to sudden release of energy and the magnitude of an earthquake is related to how much energy is released by the quake. Let's look at the Richter scale, a logarithmic function that is used to measure the magnitude of earthquakes.




**Where $A_0$ be the smallest movement that can be detected on a seismograph on general days and A be the wave movement on an particular day.**

Then Richter scale(R) measure of the magnitude of the earthquake can be find by using the formula (this has been given by Charles Richter in 1935 to measure magnitude of earthquake)

$$R = \log\left(\frac{A}{A_0}\right)$$

The intensity of an earthquake will typically measure between 2 and 10 on the Richter scale.

**Example:-**

**On a normal day the energy released by the earth crust is $8 \times 10^5$ Joule ($A_0$) and on the day of earthquake energy released $7.94 \times 10^{13}$ joule (A). find the magnitude of the earthquake using Richter Scale(R).**

**Solution:-** Now using above formula

$$R = \log_{10}\left(\frac{A}{A_0}\right) = \log_{10}\left(\frac{7.94\times10^{13}}{8\times10^5}\right)$$

$$= \log_{10}\left(9.925\times10^7\right)$$

$$= \log_{10}(9.925) + 7\log_{10}10$$

$$= (0.9967) + 7 = 7.99$$

(where $\log_{10}(9.925) = 0.9967$, you will study later in higher classes)

**Thus R = 7.99; according to the table given above it is a major earthquake causing serious damage.**

# 3. USES OF LOGARITHM TO MEASURE SOUND

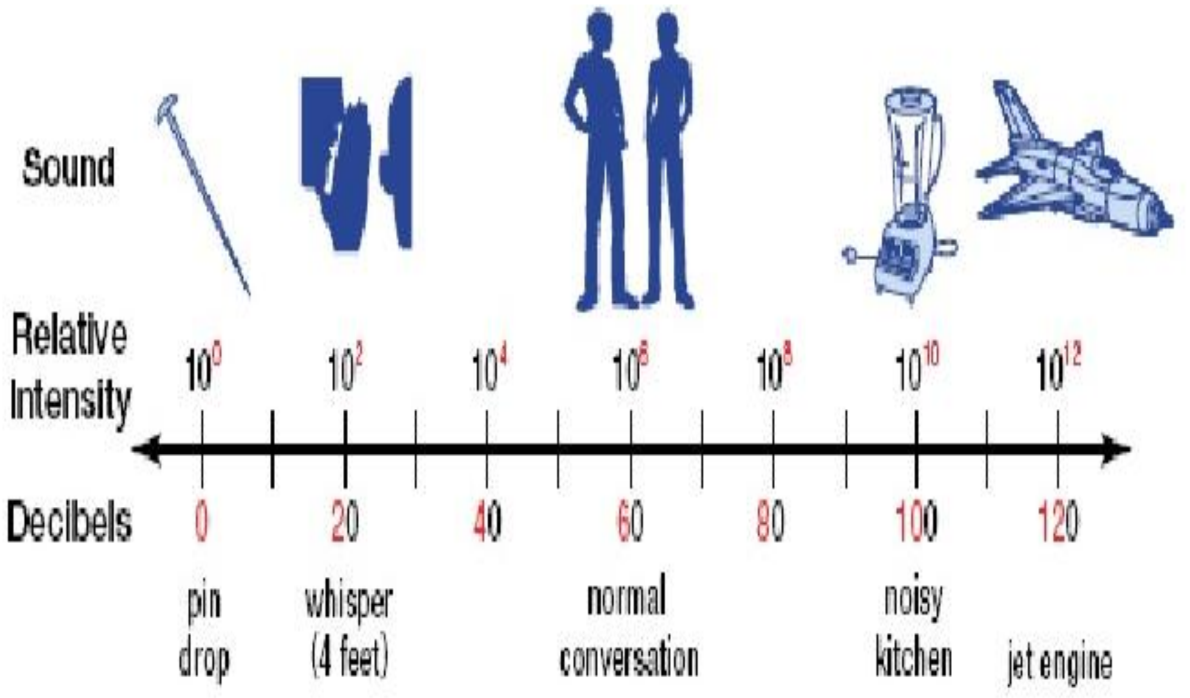
The decibel, dB, is commonly used to quantify sound levels although it is not a unit of sound, but a unit of pressure. This measure of relative sound intensity is the decibel, abbreviated dB. Because it is based on a ratio of units, the decibel is a dimensionless unit. Sound is measured in *decibel* using logarithmic. The formula looks similar to the Richter scale and is given as:

$$\text{dB} = 10\ \log_{10}\left(\frac{P}{P_0}\right) \quad ;$$

where $P$ is the power or intensity of the sound and $P_0$ is the weakest sound that the human ear can hear. We can take $P_0$ (the quietest sound) as $10^0$ in order to calculate the sound in dB

**Example:-**

**In a normal conversation between Sameer and Siya has a noise rating of 60decibels. One Jet engine, however, has a noise rating of 120 decibels.**




**The Jet engine noise is how many times more intense than the normal conversation noise?**

## Solution:-

| Normal conversation noise rating = 60 decibel = d,<br>Let h be the intensity of normal conversation | Jet engine noise rating = 120 decibel = d,<br>Let k be the intensity of Jet engine |
|---|---|
| d = 10 log $\left(\frac{P}{P_0}\right)$<br>60 = 10 log $\left(\frac{h}{P_0}\right)$<br>60/10 = log $\left(\frac{h}{P_0}\right)$<br>6 = log $\left(\frac{h}{P_0}\right)$<br>$10^6 = \frac{h}{P_0}$<br>h = $10^6$ $P_0$ --------------→ Eqn. 1 | d = 10 log $\left(\frac{P}{P_0}\right)$<br>120 = 10 log $\left(\frac{k}{P_0}\right)$<br>120/10 = log $\left(\frac{k}{P_0}\right)$<br>12 = log $\left(\frac{k}{P_0}\right)$<br>$10^{12} = \frac{k}{P_0}$<br>k = $10^{12}$ $P_0$ --------------→ Eqn. 2 |

Now dividing equation 2 by equation 1.

We get,

$$\frac{k}{h} = \frac{10^{12} P_0}{10^6 P_0}$$

$= 10^6$

***Answer: - The jet engine noises is $10^6$ times as intense as the normal conversation.***

With decibels, every increase of 10 means the sound is 10 times more intense. An increase of 20 would be 10 times more intense for the first 10, and another 10 times more intense for the second 10—so a sound that is 75 decibels is 100 times more intense than a sound that is 55 decibels!

# 4. USES OF LOG TO MEASURE HOME RANGE OF CARNIVOROUS

Sameer Grand parents lives in a village Uregi, interior area of Uttarakhand. One night a leopard enter into the village in search of food. He attacked on the grand mother of sameer. But somehow she manages to escape. Now the villagers were in fear that tiger will attack again. Old villagers know the range of leopard attack.

Do you know that this is all mathematical calculations.

It depends on the mass of the animals that fix its range of attack. The forest rangers have to fix the cages to catch the leopard.

It has been found that the home range of carnivorous (meat – eating) mammals is generally given by the formula :

$$H\,(w) = 0.11\ W^{1.36}$$

Where W is the mass of the animal in grams and H(w) is the home range in hectares (note that 1 hectare = 0.01 sq. km.)

**Example:-**

**if Leopard weighing around say 50 kg, Forest rangers want to put cages for the safety of villagers to catch the leopard. Calculate the areas that is under the risk of leopard attack.**

**Solutions:-**

Taking log both sides$H\,(w) = 0.11\ W^{1.36}$ , we get

$\log_{10} H\,(w) = \log_{10}\{\,0.11\ W^{1.36}\,\}$

$\log_{10} H(w) \quad = \log(11/\,100) + \log W^{1.36} = \log 11 - 2\log 10 + 1.36 \log 50{,}000$

$= 1.0414 - 2 + 1.36(\,4 + \log 5)$

$= 1.0414 - 2 + 1.36 \times 4.6990$

( Here we have used $\log 11 = 1.0414$ & $\log 5 = 0.6990$)

$= 1.0414 - 2 + 6.3906 = 5.4320$

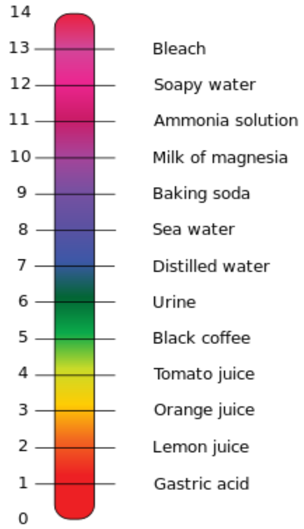
Hence$H\,(w) \quad = 10^{5.4320}$

$= 270400$ hectares $= 2704$ sq. km (approx.)

(Using logarithmic, we have $10^{5.4320} = 270400$ which you will study in higher classes).

# 5. USES OF LOGARITHM TO MEASURE PH LEVEL

The measure of acidity of a liquid is called the pH of the liquid. This is based on the amount of hydrogen ions ($H^+$) in the liquid.




The formula for pH is:

$$pH = -\log[H^+]$$

where $[H^+]$ is the concentration of hydrogen ions, given in a unit called mol/L ("moles per liter"; one mole is $6.022 \times 10^{23}$ molecules or atoms).

If pH <7 -> Solution is acidic.

If pH =7 -> Solution is neutral

If pH>7 ->Solution is basic which is also known as alkaline solution

**Key points with examples**

$\boxed{\log_a a = 1 \qquad \log_a 1 = 0}$

$\boxed{a^x = N \qquad \log_a N = x}$

$2^3 = 8 \qquad \log_2 8 = 3$

$3^4 = 81 \qquad \log_3 81 = 4$

$5^3 = 125 \qquad \log_5 125 = 3$

$10^4 = 10000 \qquad \log_{10} 10000 = 4$

$7^1 = 7 \qquad \log_7 7 = 1$

$5^0 = 1 \qquad \log_5 1 = 0$

| NUMBER | LOGARITHM |
|---|---|
| 1 | 0.00 |
| 2 | 0.30 |
| 3 | 0.48 |
| 4 | 0.60 |
| 5 | 0.70 |
| 6 | 0.78 |
| 7 | 0.85 |
| 8 | 0.90 |
| 9 | 0.95 |
| 10 | 1.00 |

Its seems to be very interesting.

Yes!!!

In all these examples we have shown you how logarithms is used in different spheres of life.

**Let's do some challenges.**

## Challenge - 1

Find the value of log 2 + $\log_{10}100$ without using a calculator

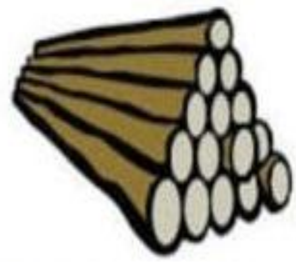
## Challenge - 2

# Complete the following table

| | | | | |
|---|---|---|---|---|
| **Exponential Form** | $2^5 = 32$ | | $3^{-2} = 1/9$ | |
| **Logarithm Form** | | $\text{Log}_{10}1000 = 3$ | | $\text{Log}_{16}4 = 1/2$ |

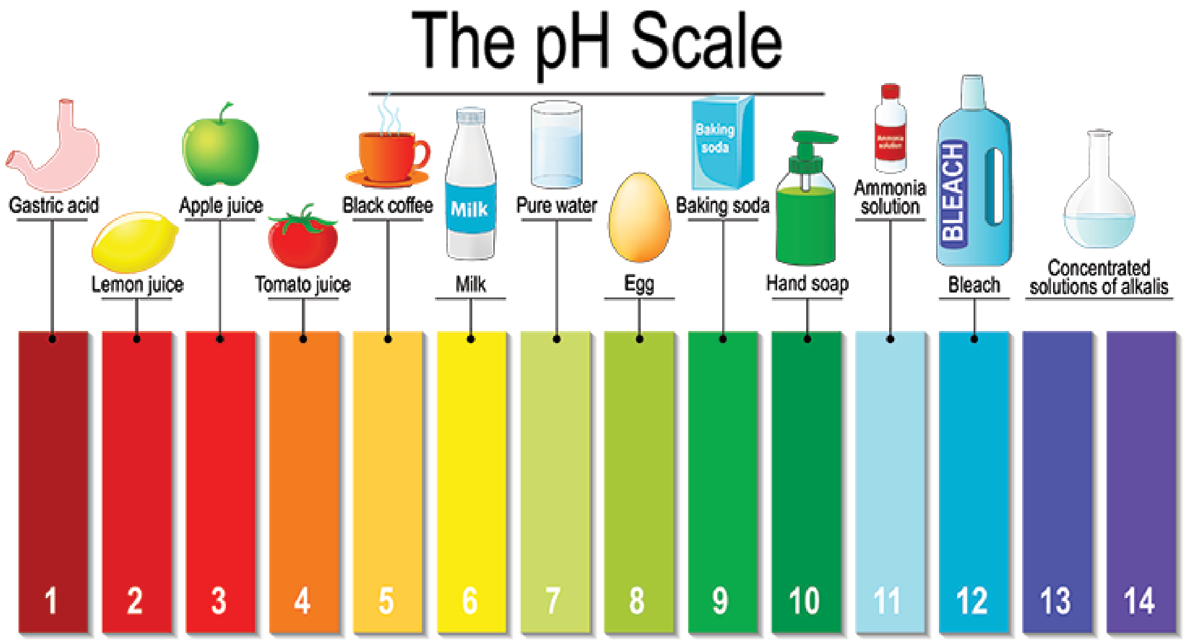
## Challenge – 3

### How much do you know?



1. What is the inverse of the exponential function?
   a. Linear function　　b. Polynomial function
   c. Quadratic function　　d. Logarithmic function
2. Which of the following is equivalent to $\log_2 16 = 4$?
   a. $2^4 = 16$　　b. $16^2 = 4$
   c. $4^2 = 16$　　d. $16^4 = 2$
3. Which of the following is equivalent to $(125)^{1/3} = 5$?
   a. $\log_{125}5 = 1/3$　　b. $\log_{1/3}5 = 125$
   c. $\log_5 125 = 1/3$　　d. $\log_5{}^{1/3} = 125$

## Challenge – 4




**a. Is black coffee acidic, basic or neutral?**
**b. Is Hand soap acidic, basic or neutral?**
**c. What is neutral solution in this figure?**
**d. ____________ is pH value of Lemon juice.**

## Challenge - 5

Sahil's parents wants to purchase a new car in 2021. Cost of car is Rs. 6,00,000/-. They want to take a loan of Rs. 50% amount of the cost price of car. If the rate of interest is 10% per annum. Find the amount they have to pay if they take loan for 4 years and 8 years.

Sahil's father asked him to calculate both the cases to decide which is better option.

## Challenge - 6

# Spread of virus

On a college campus of 5000 students, one student returns from a vacation with a contagious and long lasting flu virus. The spread of the virus is modeled by:

$$Y = \frac{5000}{1+4999(2.7)^{(-0.8t)}} : t \geq 0$$

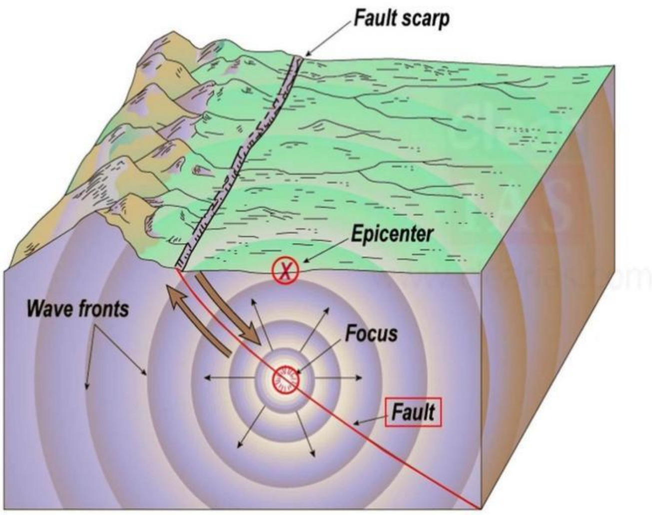
a. After 5 days how many students are infected?

b. Classes are cancelled when the number infected is 40% or more. How many days will it take for this number to be reached?

(You may use $(2.7)^4 = 53.14$ , $\log_{10} 2.7 = 0.5145$ , $\log_{10} 1.5 = 0.1761$ & $\log_{10} 4999 = 3.5228$ )

## Challenge - 7

An earthquake monitoring station measured the amplitude of the waves during a recent tremor. It measured the waves as being 100,000 times as large as $A_0$, the smallest detectable wave. How high did this earthquake measure on the Richter scale using formula $R = \log_{10}\left(\frac{A}{A_0}\right)$?




A) 1

B) 3

C) 5

D) 9

## Challenge - 8

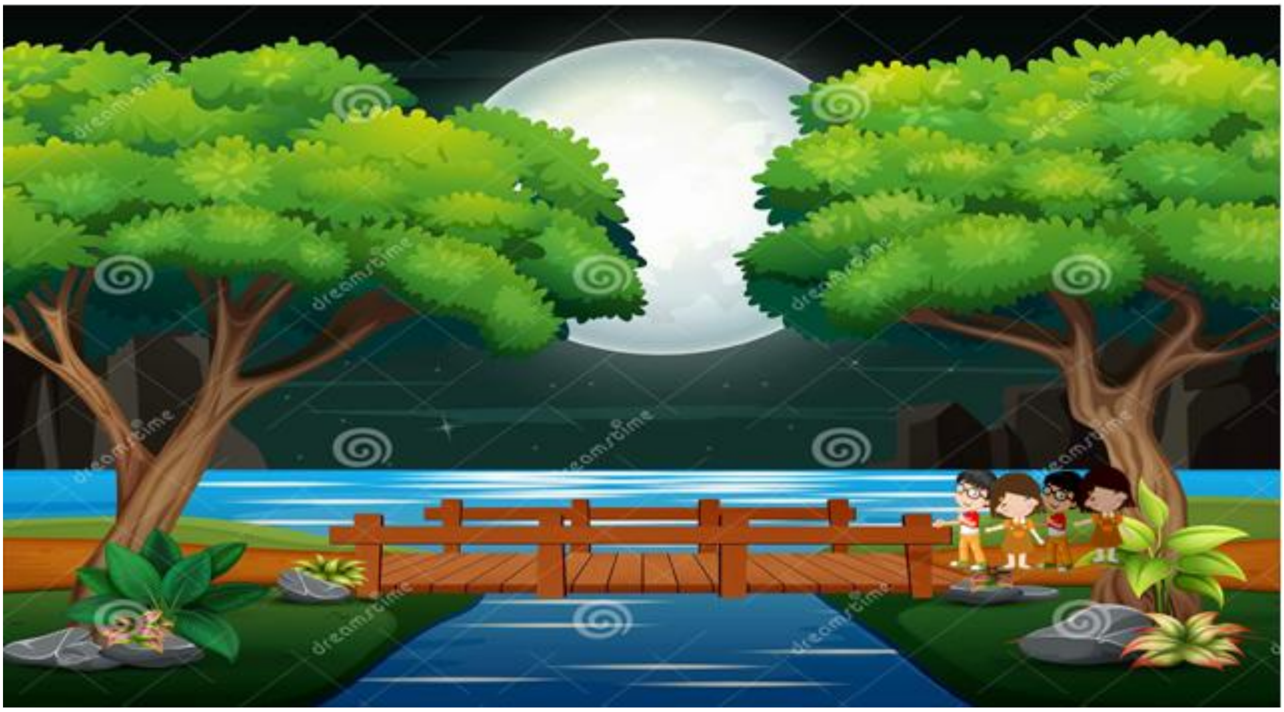
Using the world population formula $P = 6.9(1.011)^t$, where t is the number of years after 2011 and P is the world population in billions of people, estimate :


a) The population in the year 2050 to the nearest hundred million.

b) By what year will the population be double then what it was in 2011?

(You may use $\log_{10} 6.9 = 0.8388$ , $\log_{10} 1.011 = 0.0048$ , $\log_{10} 2 = 0.3010$ & $10^{1.026} = 10.6$ )

# FUN WITH MATHS



Four people come to an old bridge in the middle of the night. The bridge is rickety and can only support 2 people at a time. The people have one flashlight, which needs to be held by any group crossing the bridge because of how dark it is.

Each person can cross the bridge at a different rate: one person takes 1 minute, one person takes 2 minutes, one takes 5 minutes, and the one person takes 10 minutes. If two people are crossing the bridge together, it will take both of them the time that it takes the slower person to cross. Unfortunately, there are only 17 minutes worth of batteries left in the flashlight.

**How can the four travelers cross the bridge before time runs out?**

# MATHS IS EVERY WHERE

## MAHAभारत

सुदामा चला कृष्ण के द्वार।

मन में उड़ती उमंगों की डार।

बीस कदमों की रह गयी दूरी ।

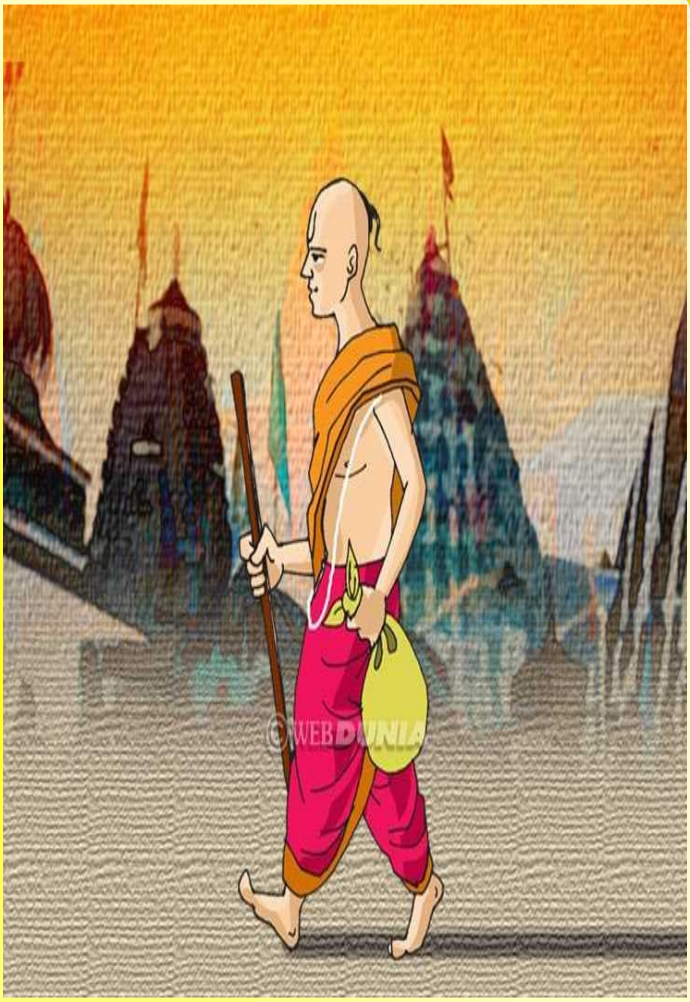
तन कहता रहने दे सुदामा,

मन कहता मिलना है जरूरी ।

तीन कदम आगे, एक चलता पीछे।

चलता है कुल कितने कदम,

चला झुकाए हुए मुख नीचे ।




**प्रश्न:- कृष्ण जी को मिलने के लिए सुदामा को कुल कितने कदम चलने पड़े?**


THINKING ABOUT THE ANSWERS, TRYING TO SOLVE IT
KYA TUMHE QUESTIONS SAMJH AAYE?
Mujhe sab samajh aa gya. These questions were very easy. Anyone can solve them if their basics are clear
Siya, hopefully you can solve all these questions now
Keep on doing Bacho.
I will share the answers next time.

# LEARNING OUTCOMES ACHIEVED

## CLASS 6

- Applies appropriate operations (addition, subtraction, multiplication and division) in order to solves problems involving large number.

## CLASS 7

- Apply algorithms to calculate rate of interest in calculation.
- Apply properties of exponential and logarithms in order to simplify problems involving multiplication and division of large number

## CLASS 8

- Observe a given context in order to apply the concept of profit and loss, discout, VAT, simple and compound interest.
- Apply the laws of exponent and logarithms to simplify a given expression.

## CLASS 9

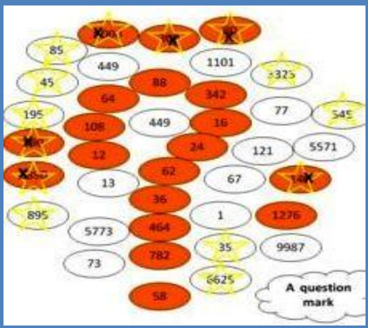
- Extend the laws of exponents andlogarithms to simplify a given expression.

## CLASS 10

- Laws of exponents studied earlier to solve problems related to real life context with the help of logarithms.

# STEP 2
# (ANSWER KEY)

Match your IQ with my answers of our previous book.

| CHALLENGE 1 | CHALLENGE 2 | CHALLENGE 3 |
|---|---|---|
|  | 4 | 3500 OR 3413<br>First deal is better |
| **CHALLENGE 4** | **CHALLENGE 5** | **CHALLENGE 6** |
| RBC BY $4.67 \times 10^{-6}$ | EQUAL = $1/3^5$ | A. $2^0$, $2^1$, $2^2$, $2^3$, $2^4$<br>B. $2^{14}$<br>C. $2^5 = 32$ |
| **CHALLENGE 7** | **CHALLENGE 8** | **CHALLENGE 9** |
| A. HAVE AN ICE DAY<br>B. I AM STUFFED | A. YES<br>B. 16,32,64,128,256,512<br>C. 2 TIMES<br>D. $2^{19}$, $2^{29}$, $2^{63}$ | A. $2^1$, $2^2$, $2^3$, $2^4$, $2^5$, $2^6$, $2^7$, $2^8$, $2^9$, $2^{10}$, $2^{11}$<br>B. $2^{19}$<br>C. 127 |
| | |  |

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