

PGT Syllabus (Concerned Subject - Mathematics)

1. SETS :

Sets and their representations. Empty set. Finite and infinite set. Equal sets. Subset of the set of real numbers especially intervals (with notations). Power set. Universal set. Venn – diagrams. Union and intersection of sets. Complement of a set.

2. RELATIONS AND FUNCTIONS:

Ordered pairs. Cartesian product of sets. Number of elements in the Cartesian product of two finite sets. Cartesian product of the reals with itself (up to $\mathbb{R} \times \mathbb{R} \times \mathbb{R}$) Definition of relation, pictorial diagram, domain, co – domain and range of a relation.

Function as a special kind of relation from one set to another. Pictorial representation of a function, domain, co – domain and range of a function.

Real value function of the real variable, domain and range of these functions.

Type of functions : constant, identity, polynomial, rational, modulus, signum and greatest integer functions with their graphs. Sum, difference, product and quotients of functions.

Types of relations : reflexive, symmetric & transitive. Equivalence relations. One to one and onto functions. Composite functions. Inverse of a function.

Groups, subgroups, normal groups

3. MATHEMATICAL INDUCTION.

Processes of proof by induction, Motivating the application of the method by looking at natural numbers as the least inductive subset of real numbers. The principle of mathematical induction and simple applications.

4. PERMUTATIONS & COMBINATIONS

Fundamental principle of counting. Factorial 'n' (i.e. n!). Permutations and combinations derivations of formulae and their connections, simple applications.

Fundamental theorem of arithmetic, divisibility in \mathbb{Z} , Congruence, Chinese remainder theorem, Euler's function.

5. BINOMIAL THEOREM

History, statement and proof of the binomial theorem for positive integral indices.

Pascal triangle, general term and middle term in the binomial expansion, simple applications.

6. LINEAR INEQUALITIES & LINEAR PROGRAMMING

Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representations on the number line. Graphical solutions of linear inequalities in two variables. Solution of system of linear inequalities in two variables - graphically.

Introduction, definition of related terminology such as constraints, objective function, optimization, different types of linear programming (L. P.) problems, mathematical formulation of L. P. problems.

Graphical method of solution for problems in two variables, feasible and infeasible solutions.

Optional feasible solutions (up to three non – trivial constraints).

8. TRIGONOMETRIC FUNCTIONS

Positive and negative angles. Measuring angles in degrees and radians and conversion of one measure to another. Definition of trigonometric functions with the help of unit circle. Sign of trigonometric functions. Expressing $\sin(x+y)$ and $\cos(x+y)$ in terms of $\sin x$, $\cos x$, $\sin y$ and $\cos y$. Deducing the identities like following : $\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$, $\cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$

$$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right), \quad \sin x - \sin y = 2 \cos\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right), \quad \cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right).$$

Identities related to $\sin 2x$, $\cos 2x$, $\tan 2x$, $\sin 3x$, $\cos 3x$ and $\tan 3x$. General solution of trigonometric equations of the type $\sin \theta = \sin \alpha$, $\cos \theta = \cos \alpha$ and $\tan \theta = \tan \alpha$.

9. INVERSE TRIGONOMETRIC FUNCTIONS

Definition, range, domain, principal value branches. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

10. DIFFERENTIAL CALCULUS & ITS APPLICATIONS

Intuitive idea of limit. Definition of derivative, Derivative as rate of change, relate it to slope of tangent of the curve. Derivative of sum, difference, product and quotient of functions.

Derivatives of polynomial and trigonometric functions.

Continuity and differentiability. Concept of exponential and logarithmic functions and their derivatives. Derivative of composite functions, chain rule. Derivatives of inverse functions, derivatives of implicit functions, logarithmic differentiation & derivative of functions expressed in parametric forms. Second order derivatives. Rolle's theorem and Lagrange's Mean Value theorems (without proof) and their geometric interpretations.

Rate of change, Increasing / decreasing functions, Tangents & normals,

Maxima & minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real – life situations).

Partial derivatives.

11. COMPLEX NUMBERS & QUADRATIC EQUATIONS.

Need for complex numbers, especially $\sqrt{-1}$, to be motivated by inability to solve every quadratic equation. Brief description of algebraic properties of complex numbers. Argand plane and polar representation of complex numbers. Statement of Fundamental Theorem of Algebra, solution of quadratic equations in the complex number system. Square root of a complex number.

Algebra of complex numbers, the complex planes., polynomials, transcendental functions such as exponential, trigonometric and hyperbolic function.

12. SEQUENCE & SERIES

Sequence and series. Arithmetic progression (A. P.), arithmetic mean (A. M.). Geometric progression (G. P.), general term of a G. P.; geometric mean (G. M.); Relation between A.M. and G.M. Sum to n terms of the special series : $\sum n$, $\sum n^2$ and $\sum n^3$.

Sequences and series of functions, uniform convergent.

13. MATRICES & DETERMINANTS

Concept of matrix , notation , order of matrix , equality of matrices , transpose of a matrix , types of matrices (zero matrix , row matrix , column matrix, square matrix , diagonal matrix , scalar matrix, identity matrix, symmetric matrix and skew – symmetric matrix).

Addition of matrices , subtraction of matrices , multiplication of a matrix with a scalar and multiplication of two matrices. Properties of matrix addition and multiplication. (non – commutativity of multiplication of matrices and existence of non – zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Properties of transpose of matrices (without proof). Invertible matrices and proof of the uniqueness of inverse, if it exists. (Here all matrices will have real entries).

Determinant of a square matrix (up to 3 x 3 matrices). Properties of determinants . Applications of determinants in finding the area of a triangle, consistency of system of equations and in finding minors, cofactors, adjoint , inverse of a square matrix. system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Eigen values and eigen vectors, Cayle – Hamilton theorem.

14. 2- DIMENSIONL GEOMETRY

Brief recall of 2 D from earlier classes. Slope of a line and angle between two lines. Various forms of equations of a line parallel to axes , point – slope form , slope – intercept form , two point form , intercept form and normal form. General equation of a line. Distance of a point from a line.

Sections of a cone : circles , ellipse , parabola , hyperbola , a point , a straight line and pair of straight lines as a degenerated case of a conic section. Standard equations and simple properties of parabola , ellipse and hyperbola, standard equation of a circle.

15. VECTORS & 3- DIMENSTIONAL GEOMEETRY

Vectors and scalars , magnitude and direction of a vector. Direction cosines / ratios of vectors. Types of vectors (equal , unit , zero , parallel and collinear vectors) , position vector of a point , negative of a vector , components of a vector, addition of vectors , multiplication of a vector by a scalar , position vector of a point dividing a line segment in a given ratio. Scalar (dot) product of vectors , projection of a vector on a line. Vector (cross) product of vectors. Scalar triple product.

Coordinates axes and coordinate planes in three dimensions. Co – ordinates of a point. Distance between two points and section formula. Direction cosines / ratios of a line joining two points.

Cartesian and vector equation of a line , coplanar and skew lines , shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines , (ii) two planes (iii) a line and a plane. Distance of a point from a plane.

16. INTEGRAL CALCULUS

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts.

Only simple integrals of the type :

$$\int \frac{dx}{x^2 \pm a^2} , \int \frac{dx}{\sqrt{x^2 \pm a^2}} , \int \frac{dx}{\sqrt{a^2 - x^2}} , \int \frac{dx}{ax^2 + bx + c} , \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

$$\int \frac{px + q}{ax^2 + bx + c} dx , \int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx , \int \sqrt{a^2 \pm x^2} dx , \int \sqrt{x^2 - a^2} dx \text{ to be evaluated.}$$

Definite integrals as a limit of a sum.

Fundamental Theorem of calculus (without proof).

Basic properties of definite integrals and evaluation of definite integrals.

Applications s in finding the area under simple curves, especially lines , area of circles / parabolas / ellipse (in standard form only) , area between the two above sais curves (the region should be

carefully identifiable
Riemann sums and Riemann integral, Improper Integrals.

17. DIFFERENTIAL EQUATIONS

Definition , order and degree , general and particular solutions of a differential equations.
Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables , homogeneous differential equations of first order and first degree. Solution of linear differential equation of the type : $\frac{dy}{dx} + P(x) y = Q(x)$, where P (x) and Q (x) are functions of x'.
Existence of uniqueness of solutions of initial value problems or the first order differential equations, singular solutions of first order differential equations.

18. STATISTICS & PROBABILITY

Measure of dispersion , mean deviation , variance and standard deviation of ungrouped / grouped data.
Analysis of frequency distributions with equal mean but different variance.
Correlation and regression analysis including Rank Method.

Random experiment , outcomes , sample space (set representation). Events , occurrence of events , 'not' & 'or' events , exhaustive events , mutually exclusive events. Axiomatic (set theoretical) probability , connections with the theories of earlier classes. Probability of an event , probability of 'not' , 'and' & 'or' events.

Multiplication theorem on probability. Conditional probability , In dependent events, Total probability , Baye's theorem , Random variable and its probability distribution , mean and variance of haphazard variable. Repeated independent (Bernoulli's) trials and Binomial distribution.
Normal distribution and Poisson distribution.
Corelation and regression analysis.

19. COMMERCIAL MATHEMATICS:

Perpetuity , Stock, shares and debenture, sinking fund, Compound interest, Income tax, GST.

20. MATHEMATICAL REASONING:

Mathematically acceptable statements. Connecting words / phrases – consolidating the understanding of “if & only if (necessary & sufficient) conditions “, “implies” , “and / or “, “implied by” , “and “ , “or” , “there exist” and their use through variety of examples related to real life and mathematics. Validating the statements involving the connecting words - difference between contradiction , converse and contrapositive.